Associate Professor Camelia OPREAN, PhD
E-mail: camelia.oprean@ulbsibiu.ro
Associate Professor Cristina TĂNĂSESCU, PhD
Associate Professor Vasile BRĂTIAN, PhD
University Lucian Blaga of Sibiu

ARE THE CAPITAL MARKETS EFFICIENT? A FRACTAL MARKET THEORY APPROACH

***Abstract**: Efficient Market Hypothesis (EMH) has attracted a considerable number of studies in empirical finance, particularly in determining the market efficiency of the capital markets. Conflicting and inconclusive outcomes have been generated by various existing studies in EMH. The purpose of the fractal market theory approach of this paper is to investigate whether the selected capital markets abide by a particular evolution pattern or the random walk hypothesis. In this paper, the Hurst exponent calculated by the R/S analysis is our measure of long range dependence in the series. As a result, we may note that the Brazilian financial market is the closest one to the random walk hypothesis, a characteristic of efficient market hypotheses. Nevertheless, the Estonian, Chinese and Romanian financial markets are the furthest ones from the random walk premise, as they observe the fractal Brownian motion.*

Keywords*: market efficiency, fractal, Hurst exponent, R/S analysis.*

JEL Classification: C5, G1

1. Introduction: market efficiency in the light of the classical EMH theories

Considerations concerning the efficiency of financial markets lay under two theories: random walk and the theory of efficient markets. The first theory, random walk, is the theory of random movement of the financial assets. Elaborated during the 6^{th} decade of the 20^{th} century, it supports the idea that the future movement of an asset is independent from past movements of assets on a market. In an informational efficient market, price movements are unpredictable, because they encompass the information and expectations of all market participants.

The second theory, which refers to the hypothesis of efficient markets, was established in the early 60s and assumes that asset markets process with great sensitivity the economic intelligence which they receive and react quickly to adjust the course of financial assets. The theory of efficient markets justifies the need of balanced markets. Fama (1970) have operationalized this hypothesis. In his famous study, which will definitively mark the theory of efficient markets, *Efficient Capital Markets: A Review of Theory and Empirical Work,* written by Fama in 1970, he gives the following definition: "A market in which prices always reflect the available information is called an efficient market". In this paper, he realizes a synthesis of previous research concerning the predictability of capital markets, the notions of *fair game* and *random walk* becoming well formulated.

The theory and models regarding capital market operation have initially developed from the assumption that these represent efficient markets. Rational agents quickly assimilate any kind of information that proves relevant to asset pricing and their output and subsequently adjusts the price in accordance with this information. By way of explanation, agents do not benefit from different comparative advantages in the process of information acquisition. To sum up, the efficient market hypothesis refers mainly to three fundamental and highly controversial concepts, as we have mentioned in the previous chapters: efficient markets, random trajectories, rational agents.

The prevailing opinion before 1970s was that the processes of evolution are deterministic and predictable, whereas nowadays these processes are considered to be rather stochastic and unpredictable, and even chaotic in most situations. Financial markets represent a potential application of chaos theory in the field of economics. The rationale for applying chaos theory to finance is that markets are non-linear dynamic systems, so that the exclusive employment of statistical models for the analysis of random walk standard data might lead to wrong or irrelevant results.

Special attention has been given to the role of chaos in the field of finance especially due to the multitude of information and the interest in identifying predictable models. Tests have also shown that the market pricing of assets, though unpredictable, will evince a particular trend. The capital market is commonly acknowledged as a *self-similar* system, in the sense that its components are either similar or even identical to the whole. Another type of *self-similar* system used in mathematics is the fractal (the shape of the resulting structure is the same irrespective of the scale of representation). The based on nonlinear dynamics thus comes into being. Recent studies, particularly after 1990, have focused mainly on developing new theory-based models, borrowed from other disciplines. Some of these models rely on the fractal theory indicating that the seemingly random evolution of the trend may be further decomposed in several similar evolution periods so that the initial series enables modeling. The application of fractal methods – Fractal Market Hypothesis (FMH) [8], [9] – allows us to explore the chaotic nature of time series [10]. This theory is a rendering of efficient markets where the focus is placed on market stability however, instead of market efficiency. Thus, it takes into account the everyday market randomness as well as market anomalies.

The paper is structured as follows. Section 2 describes the methodology used in view of attaining the goal of this paper, the methods we used for the Hurst exponent estimation. In Section 3, we describe the data and Section 4 covers the empirical results and discusses the implications whether the selected markets are subject to a specific evolution pattern or the random walk hypothesis. Section 5 concludes.

2. Methodology

2.1. The R/S fractal analysis method – rescaled range analysis

This method, applicable to the study of financial markets, relies on the concept of the Hurst exponent. It was introduced by English hydrologist H.E. Hurst in 1951, based on Einstein's contributions regarding Brownian motion of physical particles, to deal with the problem of reservoir control near Nile River Dam. Hurst (1965) proved that the dynamics of many natural phenomena is given by a randomly changed law. If the data series were perfectly random then these value series would increase simultaneously with the square-root of time enhancement (the $\sqrt{T}$ rule). Hurst introduced the non-dimensional ratio, dividing the data series to the standard deviation of observations to the average, in view of enabling comparison of data selected at different time moments. The result was a redimensioning of the scale hence the method is also called the *R/S* rescaled range analysis. R/S analysis in economy was introduced by Mandelbrot (1972), who argued that this methodology was superior to the autocorrelation, the variance analysis and to the spectral analysis.

In this paper, the Hurst exponent calculated by the R/S analysis is our measure of long range dependence in the series. The R/S method (range / standard deviation) requires an initial dynamic series, standing for the evolution of a natural phenomenon or process.

Let P_t be the price of a stock on a time t and r_t be the logarithmic return denoted by the first difference of logarithmic values of daily prices [7]:

$$r_t = d\log(P_t) = \log(P_t) - \log(P_{t-1}) \qquad (1)$$

As financial time series display high degree of non-stationarity, it is a common practice to work with first differenced series than with the original series. Therefore in the first step, we reduce non-stationarity by converting the original series to a returns series taking logarithm returns from successive values of the series.

The R/S statistic is the range of partial sums of deviations of times series from its mean, rescaled by its standard deviation. The entire data series is divided into several contiguous sub-periods each having *n*-observations and define each sub-period as I_a, $a = 1, 2, \ldots, k$, so $k \cdot n = N$. For each sub-period, the average value of the sub-period is determined. Starting from this assumption, we may determine the following dimension (considering that the series is divided into *a* sub-periods of *n* range):

$$X_{n,a} = \sum_{i=1}^{n} (r_i - \overline{r_{n,a}}) \tag{2}$$

where $X_{n,a}$ represents the cumulative deviation for each I_a sub-period; r_i represents the *i* component of the dynamic series; $r_{n,a}$ represents the r_i value average on every I_a sub-period.

The range (R) of the cumulative trend adjusted return series for each sub-period I_a is measured by taking differences of maximum and minimum values of $X_{n,a}$:

$$R_{n,a} = Max(X_{n,a}) - Min(X_{n,a}) \tag{3}$$

Hurst divided the *R* value to the standard deviation of initial observations for each I_a sub-interval, in view of comparing various dynamic series, for instance:

$$S_{n,a} = \sqrt{\frac{\sum_{i=1}^{n} (r_i - \overline{r_{n,a}})^2}{n}} \tag{4}$$

For every I_a sub-interval, the ratio given below is further determined:

$$(R/S)_{n,a} = \frac{Max(X_{n,a}) - Min(X_{n,a})}{\sqrt{\frac{\sum_{i=1}^{n} (r_i - \overline{r_{n,a}})^2}{n}}} \tag{5}$$

As there are many contiguous sub-periods, the average R/S value of full series is calculated by averageing R/S values of all individual sub-periods:

$$(R/S)_n = \frac{1}{k} \cdot \sum_{a=1}^{k} (R/S)_{n,a} \tag{6}$$

The n length is increased and the procedure is repeated until $n = (N-1)/2$.

Hurst noticed that this ratio increases as the number of observations in the initial data series enhances. If the data series were perfectly random, then the ratio would increase proportionally with the square root of the number of observations $\sqrt{N}$. Brownian motion is the primary model for a random walk process. Einstein (1908) found the distance a particle covers increases with respect to time according to the following relation:

$$R = T^{0.5} \tag{7}$$

where R is the distance covered by the particle in time T (see [8], [9]).

We can use equation (7) only if the time series we are considering is independent of increasing values of T. To take into account the fact that economic time series systems are not independent with respect to time, Hurst (1965) found a more general form of equation (7). In order to measure the R/S ratio enhancement in dependence on the phenomenon observation time, Hurst employed the following relation:

$$(R/S)_n = c \cdot n^H \tag{8}$$

where c is a proportionality constant and H represents the Hurst exponent.

Afterwards, the value of Hurst exponent (H) is established by means of regression:

$$\log(R/S)_n = \log(c \cdot n^H) = \log(c) + H \cdot \log(n) \tag{9}$$

2.2. Hurst exponent analysis

This paper computes the Hurst statistics by means of the aforementioned R/S method (apart from the classical estimation, a different methodology shall also be used (i.e. [3]) in order to calculate the Hurst exponent) to identify the existence of long-term linear dependence in the stock market volatility. In this framework, we adopt 16 fixed-sized windows of n observations to compute the extent of inefficiency (n = 10, 20, 30, 40, 60, 80, 100, 120, 200, 240, 300, 400, 500, 600, 800, 1200).

a. Long-term correlation analysis

In the case of a time series, whenever the Hurst exponent is other than 0.5, correspondingly the breakeven point has no common distribution, hence any time series observation is no longer independent. The recent observations are attached to the “memory” of previous observations, also called long-term memory character. Mandelbrot introduced a coefficient for measuring the correlation (between increases occurring at future moments to increases at previous time), C_n, with the following significance in correlation with the Hurst exponent:

$$C_n = 2^{(2H-1)} - 1 \tag{10}$$

where C_n represents the correlation coefficient evinced throughout the time range subject to analysis.

The relation between fractal dimension D (used to identify whether the system is at random) and Hurst exponent is: $D = 1/H$ [5]. The random walk has a fractal dimension of 1/0.5 = 2. Therefore, it fully covers the entire space. If we also take into consideration this relation, three different values of Hurst exponent can tell three different types of time series:

- the Hurst exponent equals 0.5 the series is no longer correlated, and the events are perfectly random ($C = 0$, $D = 1.5$). It is standard Brownian motion. In other words, the present cannot influence the future. The event-related probability density function may be the common one or any other probability distribution. Fractional Brownian motion is a well-known stochastic process where the second order moments of the increments scale as:

$$E\left\{\left[Y(t_i) - Y(t_{i-1})\right]^2\right\} \propto \left|t_i - t_{i-1}\right|^{2H} \tag{11}$$

with $H \in [0, 1]$. The Brownian motion is then the particular case where $H = ½$.

- the correlation coefficient is less than zero, $D > 1.5$ (provided that $0 \le H < 0.5$), therefore the series is negatively correlated. This type of dynamic series is the *antipersistent* fractal Brownian motion (a characteristic of self-adjustment systems), so that an increase in the series term will most likely entail its decrease, at the next moment in time. The intensity of antipersistent behavior depends on H`s closeness to zero. If $H = 0$, then $C = -0.5$ which makes, under these circumstances, the series highly volatile and subject to frequent changes. Given that the system covers a smaller distance compared to the random movement, its evolution changes its sign more often than a random process;
- the C_n correlation coefficient is greater than zero, $D < 1.5$ (whenever $0.5 < H \le 1$), therefore the series is positively correlated and we thus encounter a persistent dynamics, i.e. if a value increased (decreased) previously, then very likely it will continue increasing (decreasing). The intensity of persistent behavior depends on the H value: whenever H tends to 1, then C tends to 1, too. Persistent dynamic series are fractal, since they are actualy functional Brownian motions with a correlation among events in time.

To sum up, for any t_1 and t_2 moments, if:

- $H = ½$ we have a *normal Brownian motion;*
- otherwise, there is a *fractal Brownian motion.*

The difference between the two motions is that the fractal Brownian motion evinces correlations on an infinitely large scale.

b. Statistic V_n

Statistic V_n was originally used to test the stability of R/S analysis method [11]. Peters expanded it to accurately measure the critical point *n* and average cycle length:

$$V_n = \frac{(R/S)_n}{\sqrt{n}} \tag{12}$$

For the time series of independent stochastic process, the curve of V_n concerning log(*n*) is a straight line. If the sequence possesses a state persistence, i.e. when $0.5 < H < 1$, V_n concerning log(*n*) is upward raked; if the sequence has an inverse state persistence, i.e. when $0 < H < 0.5$, V_n concerning log(*n*) is sloped downward.

c. Significance Test and the expectation value of H

The greatest fault of the early-period R/S analysis is the lack of significance test. As to random walk hypothesis, $H = 0.5$ is a gradual process. When the study sample is limited, the value of *H* of random sequence always deviates from 0.5. At this time the expectation value of *H* can be derived from the following formula (Peters, 1994):

$$E(R/S)_n = \frac{n-0.5}{n} \cdot \frac{1}{\sqrt{0.5 \cdot \pi \cdot n}} \cdot \sum_{i=1}^{n-1} \sqrt{\frac{n-i}{i}} \tag{13}$$

From the following regression analysis we can get the value of *E(H)*:

$$\log E(R/S)_n = \log(c) + E(H) \cdot \log(n) \tag{14}$$

R/S_n is the random variable of normal distribution, thus *E(H)* also follows the normal distribution, expectation variance $Var(H)_n = 1/N$, *N* is the total sample capacity. The null hypothesis of this significance test is H_0: $H = E(H)$ that is the

Gaussian process of random walk; the alternative hypothesis H_1: $H \neq E(H)$ that sequence is biased random walk and has persistence and memory effect. The statistic t of hypothesis test is:

$$t = \frac{H - E(H)}{\sqrt{1/N}} \tag{15}$$

The loss of accuracy of R/S analysis in some specific cases (problems to study long range dependence when series is not large enough) has been remarked by some authors ([3], [9], [12] and others).

Following Weron (2002), once $E(R/S)_n$ is calculated, the Hurst exponent H will be 0.5 plus the slope of $(R/S)_n - E(R/S)_n$. However, if we calculate this modified R/S analysis in this way, results show a Hurst exponent, for some random series, with values higher than 1, which makes no sense. For this reason, we have followed a different procedure [3] that lies in adding other step to the classical R/S analysis which consist in calculating:

$$\log H_n = \log R/S_n - \log E(R/S)_n + \log n/2 \tag{16}$$

Then find H by linear regression on:

$$\log H_n = \log c + H \cdot \log n \tag{17}$$

3. Data

The data of this survey represent daily stock market quotes of the most traded indices in the eight emergent capital market countries, four UE countries and the four BRIC countries (BET for Romania, OMX Tallin for Estonia, PX for the Czech Republic, BUX for Hungary, BOVESPA for Brazil, SENSEX for India, RTSI for Russia, and the Shanghai Composite Index for China). The indices are selected as they can best reflect and exhaustively capture all events on a market.

Data has been mainly collected from stock exchange own websites, showing the indices transaction. We have collected daily values over a ten-year time span, between October 2002 – May 2014, in order to reach conclusions based on thorough and accurate data; the time span thus enabled a proper modeling of the phenomena occurring on the respective capital markets. The data sample consists of an average of 2500 daily values of each index price separately.

4. Results

The aim of the tests we are going to perform is to identify whether the selected markets are subject to a specific evolution pattern or the random walk hypothesis.

4.1. Tests for the hypothesis of normality and independence of instantaneous returns of indices

Standard descriptive statistics applicable to the selected index series are produced in table 1. It shows means, medians, standard deviations, skewness, kurtosis, Jarque-Bera test and its probability for the daily returns obtained from the first difference of the series.

The lowest level of returns is to be noticed in India with a value of -0.076% and in Brazil (-0.07%) whereas the highest level occurs in Russia (0.057%). The market risk (systematic risk) is measured by the standard deviation which is higher in Russia, this capital market witnessing a high volatility. The lowered risk goes to the financial Estonian market.

A negative Skewness indicates that the tail on the left side of the probability density function is longer than than the right side and the bulk of the values possibly including the median, tie to the right of the mean. The BET, BUX, PX and RTSI series are asymmetrical to the left side because the skewness is negative. Likewise, the OMX, BOVESPA, SENSEX and Shanghai Composite Index are asymmetrical to the right side because skewness is positive. Kurtosis indicates that the series have a higher variance than the most specific normal distribution (k=3) and a leptokurtic distribution of indices returns. Due to the probability value of zero discussed by our study, the null hypothesis for the normal distribution of indices is rejected.

Table 1. Descriptive statistics of the index series

Index	Mean %	Median %	St. dev. %	Skewness	Kurtosis	JB	Probability
BET	0.043	0.062	1.762	(0.61)	10.09	5396.45	0.000
PX	0.032	0.088	1.550	(0.57)	16.93	20487.95	0.000
OMX Tallinn	0.046	0.000	1.165	0.17	12.57	10505.51	0.000
BUX	0.039	0.062	1.696	(0.10)	9,16	3979.17	0.000
BOVESPA	(0.070)	(0.135)	1.853	0.09	8.10	2689.63	0.000
RTS	0.057	0.208	2.247	(0.49)	13.98	12635.21	0.000
SENSEX	(0.076)	(0.133)	1.642	0.08	10.75	6198.35	0.000
SSE Composite	(0.012)	(0.008)	1.658	0.24	6.62	1403.00	0.000

Also, we cannot conclude that the series of returns have normal distributions using the Jarque Bera test. Most markets exhibit deviations (volatility) from normality. Due to the fact that returns are correlated and do not have a normal distribution, we reject the hypothesis that supports the idea that these time series are yielding a random model, therefore, we question the existence of a weaker form of informational efficiency for the eight emergent capital markets.

Emergent markets are defined by a non-linear behavior of information in terms of stock prices. Consequently, we applied the BDS independence test in order to determine the time dependence throughout a given chronological sequence. The assumption that a system represents a nonlinear process is one of the cahos theory premises that has got the attention of most economists. The BDS test developed by Brock, Dechert, Scheinkman and Le Baron (1996) is arguably the most popular test for nonlinearity. It does not directly test chaos (which would be impossible) but only nonlinearity. If evidence of nonlinearity should be found in time series, this shows for at least a short period of time, that predictions may be improved by alternating from a linear to a nonlinear strategy.

Moreover, it can be used as a mis-specification test when applied to the residuals from a fitted model. Evidence of nonlinear dependence of returns may create doubts on the informational efficiency of financial markets. In particular, when applied to the residuals from a fitted linear time series model, the BDS test can be used to detect remaining dependence and the presence of omitted nonlinear structure.

The null hypothesis (H_0) of an identical and independent distribution is tested. If the null hypothesis H_0 is accepted, the original linear model cannot be rejected (accepted). If H_0 is rejected, the fitted linear model is mis-specified and in this sense, it can also be regarded as a test for non-linearity. If the BDS value is statistically higher than 1.96 (2.58) then H_0 is rejected at a 5% significance level.

However, the BDS test has power against a wide range of linear and nonlinear alternatives and it is applied in most cases to the residuals of a linear model (we applied it on the residuals of the AR(1) model or GARCH type). This step is regarded as a type of pre-filtering of data, making possible the observation of any potential determinism of the analysed data, except for the one described by a linear or a GARCH type process. Brock et al. (1997) recommends an ε distance threshold between 1 and 3 halves of a standard deviation for an increased testing power. The analysis is performed for the embedding dimensions $\varepsilon/\sigma = 1$ and $\varepsilon/\sigma = 2$ (and is presented below in the table 2):

Table 2. BDS statistical test

	BDS statistic, $\varepsilon/\sigma = 1$							
dimension	BET	BUX	OMX	PX	BOVESPA	RTS	SENSEX	SHANGHAI
2	0.037297	0.016510	0.039452	0.026454	0.006932	0.023988	0.022707	0.009618
3	0.063036	0.027507	0.067514	0.048955	0.014309	0.045526	0.040908	0.021200
4	0.075473	0.032011	0.081820	0.059431	0.018055	0.054914	0.049526	0.026310
5	0.077386	0.031927	0.085954	0.062799	0.018738	0.056800	0.051592	0.026408

The results confirm the rejection of a null hypothesis H_0 and of an identical and independent distribution, nonlinearity of all series of indices is therefore considered to be checked.

4.2. Statically analysis of Hurst exponent

Based on the aforementioned method, we have obtained the following results for the indicators of the eight emergent financial markets, presented in the table 3, which gives a classification of countries subject to analysis on the basis of the Hurst exponent value:

Table 3. Indices classification according to the value of Hurst exponent

Classification	Index	Hurst exponent	Correlation coefficient C	Fractal Dimension D
8	BOVESPA (Brazil)	**0.583565**	0.49854769	1.713605
5	RTS (Russia)	**0.676366**	0.59810921	1.478489
7	SENSEX (India)	**0.647230**	0.56615826	1.545046
2	SHANGHAI_C_I (China)	**0.709749**	0.63551954	1.408949
3	BET (Romania)	**0.709547**	0.63529056	1.40935
6	BUX (Hungary)	**0.662280**	0.58258172	1.509935
4	PX (Czech Republic)	**0.679205**	0.60125714	1.47231
1	OMX Tallinn (Estonia)	**0.791595**	0.73098713	1.263272

Subsequent to the calculation of the Hurst exponent for the analysed stock-exchange indices, we may notice, as an overall trend, that all rate series are persistent, since the value of the Hurst exponent ranges between 0.5 and 1.

Therefore, the value of correlation coefficients is also more than zero, so that the series are positively correlated. Hence, if a term in the sequence has increased, it will most probably increase, at the next moment in time. All stock quote sequences undergo a fractal Brownian motion, different than the common random walk. The intensity of the persistent behavior is dependent on the *H*`s closeness to one. Given this correlation of events, the probability that two events should succeed each other is no longer 0.5. The Hurst exponent represents the probability that two similar events should occur consecutively. This is a fractal series, since each occurrence of an event is no longer equally probable, unlike perfectly random dynamic series.

Further to the classification performed in keeping with the Hurst exponent and presented in the graph above, we may note that the Brazilian financial market is the closest one to the random walk hypothesis, a characteristic of efficient market hypotheses. Nevertheless, the Estonian, Chinese and Romanian financial markets are the furthest ones from the random walk premise, as they observe the fractal Brownian motion.

The *R/S* analysis shows the fractal distribution of short-term market performance distribution, which requires a self-similar structure. This hypothesis explains the structure by means of multiple investment horizons of investors. That is, the presence of investors who choose to invest on different time spans at any time scale will make the distribution probability time-scale independent, which can definitely be called a characteristic of a fractal distribution.

Whenever, the application of the Hurst exponent to the time sequences testifies that the series are not independent, hence they do not meet the randomness criterion, we may conclude that each term of the dynamic sequence includes a long-term memory of past events, instead of a short-term memory from one term to another. Recent events have had a greater impact upon current observations, whereas previous events tend to have a less significant impact, however they still retain a residual influence [6].

4.3. Statistic V_n

Furthermore, *V-statistic* is also used for enhancing signals or bumps in the graph entailed by its cyclic behavior. When the graphic shape of V_n changes, it produces the sudden mutation, and the long-term memory also disappears. Therefore, we can see directly the influence time boundary of the value of time series at a certain moment on the future values with the relation curve of V_n concerning log(n). For the eight indices included in the analysis, the evolution of V_n statistic and log R/S according to log(n) shows that in all situations, the time series is defined by a state persistence (when $0.5 < H < 1$), because the sequence V_n concerning log(n) is upward raked. In the case of independent stochastic process, the curve of V_n concerning log(n) would be a straight line [11].

Table 4. Values of *t* statistic

	BOVESPA	RTS	SENSEX	Shanghai_C_I	PX	OMX Tallinn	BUX	BET
***t* statistic**	0.86054	5.472665*	4.024633*	7.13177*	5.613761*	11.19944*	4.772604*	7.121731*

*significant for a significance level of 0.1%

As previously mentioned, the role of *V-statistic* is to enhance the bumps in the graph, occurring as a result of cyclic behavior. For instance, the BOVESPA graph evinces a rising peak in the graph about 300 working days, about 15 months, indicating an average cycle for this particular length. The second graph, *log(R/S)*, provides a graph representation influenced by the Brownian motion. Therefore, the market shall evince a fractional Brownian motion, in the long-term behavior, rather than a cyclical one. Since the Hurst exponent is an indicator of the system memory and considering that cycles evince an ill-defined Hurst exponent, the cycle stages are an indicator of the system memory length.

4.4. Significance Test and the expectation value of *H*

The greatest fault of the early-period R/S analysis is the lack of significance test. R/S_n is the random variable of normal distribution, thus $E(H)$ (from the regression analysis: $\log E(R/S)n = \log c + E(H) * \log n$ we got the value of $E(H) = 0.566250$) also follows the normal distribution, expectation variance $Var(H)_n = 1/N$, N is the total sample capacity. The hypothesis to test are:

- the null hypothesis of this significance test is H_0: $H = E(H)$ that is the Gaussian process of random walk;
- the alternative hypothesis H_1: $H \neq E(H)$ that sequence is biased random walk and has persistence and memory effect.

The *t* statistic of hypothesis test is calculated in the table 4. Since *t* statistic for all indices, except BOVESPA, is superior to the corresponding table value ($\alpha = 0.001$), we can say with a 0.1% error risk that the null hypothesis is rejected, H is significantly different from $E(H)$, so we accept the alternative hypothesis H_1, that the indices data are biased random walk and have persistence and memory effect. The null hypothesis of this significance test is accepted only in the case of BOVESPA, so this is the closest to the Gaussian process of random walk.

Furthermore, we shall implement a different methodology (GSP - Granero, Segovia, Pérez, 2008) for calculating the Hurst exponent, which lies in adding other step to the classical R/S analysis which consists in calculating:

$$\log H_n = \log R/S_n - \log E(R/S)_n + \log n/2 \quad (18)$$

Then find H (estimated in the table 6) by linear regression on:

$$\log H_n = \log c + H \cdot \log n \quad (19)$$

The striking feature is that we have found a similar classification by the both measures. Table 5 shows that a different methodology of calculating the Hurst exponent (GSP) almost fully observes the trend evinced by the classical Hurst calculation. The only change is in the situation of the capital market with the closest evolution to the random walk, i.e. India, unlike Brazil (in the case of classical Hurst methodology).

Table 5. Estimating the Hurst exponent according to the GSP methodology

Log *N*	**Log *Hn***							
	BOVESPA (Brazil)	RTS (Russia)	SENSEX (India)	Shanghai_C_I (China)	BUX (Hungary)	PX (Czech Republic)	OMX Tallinn (Estonia)	BET (Romania)
2.302585	1.22593	1.26040	1.26218	1.21317	1.22395	1.22124	1.27734	1.27786
2.995732	1.08354	1.12182	1.17077	1.17982	1.13311	1.11032	1.13991	1.24145
3.401197	1.27803	1.40545	1.32388	1.31495	1.40228	1.40461	1.35116	1.42248
3.688879	1.49484	1.56270	1.52453	1.45289	1.58010	1.56843	1.53571	1.53141
4.094345	1.77105	1.81907	1.80532	1.73776	1.77519	1.79381	1.83553	1.77103
4.382027	1.95152	2.02848	2.02356	1.89484	1.90952	1.91567	2.05252	1.87144
4.60517	2.20149	2.23663	2.30842	2.30540	1.76692	1.92975	2.17674	1.98491
4.787492	2.37404	2.50173	2.55500	2.40086	2.14458	2.23092	2.55667	2.20278
5.298317	2.33291	2.41858	2.32680	2.44566	2.42379	2.43610	2.55630	2.52975
5.480639	2.74825	3.01098	2.90444	2.76714	2.78919	3.03101	3.40021	2.99606
5.703782	3.28008	3.24739	2.95366	3.13233	3.03686	3.24966	3.46969	3.18668
5.991465	3.00052	3.10025	3.08046	3.04478	2.96312	3.06020	3.39837	3.17012
6.214608	3.13655	3.33347	3.13023	3.35352	3.30335	3.29847	3.57526	3.38021
6.39693	3.20205	3.36945	3.34232	3.32532	3.25902	3.30531	3.77145	3.45862
6.684612	3.40654	3.53639	3.49695	3.60800	3.48178	3.50372	3.93069	3.68241
7.090077	3.57655	3.70409	3.60550	4.04631	3.66819	3.68040	4.19164	3.92655
Hurst GSP	0.586972	0.61011	0.58098	0.643499	0.596030	0.612956	0.72534	0.643297
classification	**7**	**5**	**8**	**2**	**6**	**4**	**1**	**3**
Hurst classic	0.583565	0.6763	0.64723	0.709749	0.662280	0.679205	0.79159	0.709547
classification	**8**	**5**	**7**	**2**	**6**	**4**	**1**	**3**

5. Conclusions

An application of normality and independence tests on the logarithmic profitability of the index series, shows that almost all markets evince significant deviations from normality. The time series composed of stock indices also evinces nonlinearity. The existence of a poor informational efficiency for the eight emergent capital markets is highly questionable.

Subsequent to the classical calculation of the Hurst exponent for the stock indices subject to analysis, we have noticed that all trends are persistent, since the value of the Hurst exponent ranges between 0.5 and 1. Therefore, the value of correlation coefficients is above zero, so that the series are positively correlated. According to the Hurst exponent value, the Brazilian financial market is closest to the random walk hypothesis, specific to efficient market hypothesis. On the other hand, the financial market from Estonia, China and Romania prove to be the furthest from the random walk hypothesis, characterized by the highest degree of fractal Brownian motion.

A different methodology of calculating the Hurst exponent (GSP) almost fully observes the trend shown by the classical Hurst calculation, where only case of the capital market whose evolution is closest to the random walk (i.e. India, unlike Brazil – as in the case of the classical Hurst methodology) is slightly changed.

To sum up, the tests performed in this paper indicate that the yields of these emergent stock markets is a persistent series submitting to fractal distribution.

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